

The complex relation between production and scattering amplitudes

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Abstract

The unitarity relation $\Im m(A) = T^* A$ is derived for a three-body production amplitude A that consists of a complex linear combination of elements of the two-body scattering amplitude T . We conclude that the unitarity relation does not impose a realness condition on the coefficients in the expansion of A in terms of T .

Under the spectator assumption, we deduced in Ref. [1] that the three-particle production amplitude A consists of a complex linear combination of elastic and inelastic matrix elements of the two-body scattering amplitude T . Furthermore, in Ref. [2] we showed that such a two-particle production amplitude can reasonably describe experiment in an energy region where no additional resonances from possible rescattering with the spectator particle exist. Moreover, no need to treat a sizable fraction of the experimental signal as background was noticed.

The result of Ref. [1] agrees to some extent with the expression proposed in Refs. [3, 4]. Like in our Ref. [1], the authors of Ref. [4] based their ansatz on the OZI rule [5] and the spectator picture, finding that the production amplitude can be written as a linear combination of the elastic and inelastic two-body scattering amplitudes, with coefficients that do not carry any singularities, but are rather supposed to depend smoothly on the total CM energy of the system.

However, Ref. [4] concluded from the unitarity relation

$$\Im m(A) = T^* A \quad (1)$$

that the production amplitude must be given by a *real* linear combination of the elements of the transition matrix. A similar conclusion, based on a K -matrix parametrisation, can be found in

Ref. [6]. In contrast, we arrive at a different conclusion, namely that the coefficients must be *complex*, in agreement with experiment [7–9] as well as with the work of the Ishidas [10, 11].

Relation (1), which can also be found in Ref. [12], basically stems from the operator relations $AV = (1 + TG)V = V + TGV = T$, the symmetry of T , the realness of V , and the unitarity of $1 + 2iT$, which gives $\Im m(A)V = \Im m(AV) = \Im m(T) = T^*T = T^*AV$. This leads, for non-singular potentials V , to relation (1).

Now we shall show that A and T satisfy the unitarity relation (1) *despite* the complexness of the coefficients in the expansion of A in terms of T . Thereto, we are going to strip the expressions of Ref. [1] of all details which might obscure the simplicity of our arguments. Hence, let $Z_k(E)$ ($k = 1, 2, \dots, n$) represent a vector of complex non-singular expressions¹, being smooth functions of the energy E , where n represents the number of coupled scattering channels under consideration, and let the relation between A and T be given by²

$$A_k = \Re e(Z_k) + i \sum_{\ell} Z_{\ell} T_{k\ell} . \quad (2)$$

We then find for the imaginary part of the production amplitude

$$\Im m(A_k) = \frac{1}{2i} (A_k - A_k^*) = \sum_{\ell} \{ \Re e(Z_{\ell}) \Re e(T_{k\ell}) - \Im m(Z_{\ell}) \Im m(T_{k\ell}) \} . \quad (3)$$

Next, we substitute on the right-hand side of Eq. (3) the identity $\Re e(T_{k\ell}) = T_{k\ell}^* + i\Im m(T_{k\ell})$, and furthermore insert the unitarity condition for T , *i.e.*, $\Im m(T_{k\ell}) = \sum_{\ell'} T_{\ell'\ell} T_{k\ell'}^*$, so as to obtain

$$\begin{aligned} \Im m(A_k) &= \sum_{\ell} \{ \Re e(Z_{\ell}) (T_{k\ell}^* + i\Im m(T_{k\ell})) - \Im m(Z_{\ell}) \Im m(T_{k\ell}) \} \\ &= \sum_{\ell} \left\{ \Re e(Z_{\ell}) T_{k\ell}^* + i Z_{\ell} \sum_{\ell'} T_{\ell'\ell} T_{k\ell'}^* \right\} . \end{aligned} \quad (4)$$

Finally, we interchange ℓ and ℓ' in the second term on the right-hand side of Eq. (4), leaving us, also using Eq. (2), with

$$\Im m(A_k) = \sum_{\ell} T_{k\ell}^* \left\{ \Re e(Z_{\ell}) + i \sum_{\ell'} Z_{\ell'} T_{\ell\ell'} \right\} = \sum_{\ell} T_{k\ell}^* A_{\ell} . \quad (5)$$

This completes the proof that A , as defined in Eq. (2), satisfies the unitarity condition (1). Consequently, relation (1) does not impose a realness condition on the coefficients in Eq. (2).

The first term on the right-hand side of relation (2) was not considered in Refs. [3, 4, 6]. However, in the works of Graves-Morris [13] and Aitchison & collaborators [14–16], the possible existence of an additional real contribution was anticipated. In Refs. [1, 2], this follows straightforwardly from the reasonable assumption that a produced meson pair originates from an initial $q\bar{q}$ pair. As a consequence, the observed phenomenological necessity [9] to employ complex coefficients in experimental analyses of production processes does *not* allow by itself to draw conclusions on the inevitability of including rescattering diagrams with the spectator particle in theoretical approaches.

¹In Appendix A we give the precise relation between the expressions used in Ref. [1] and Z_k .

²Note that for $\Re e(Z_k) = 0$ one obtains an expansion with real coefficients, as in Refs. [3, 4].

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A Precise definition of $Z_k(E)$

In Ref. [1] we discussed the partial-wave expansion of the amplitudes for two-meson production — together with a spectator particle — and scattering, assuming $q\bar{q}$ pair creation. Hence, the coefficients bear reference to the partial wave ℓ and the flavor content α of the quark pair. We obtained [1] the following relation between production and scattering partial-wave amplitudes:

$$A_{\alpha i}^{(\ell)} = g_{\alpha i} j_{\ell}(p_i r_0) \sqrt{\mu_i p_i} + i \sum_{\nu} g_{\alpha \nu} \sqrt{\mu_{\nu} p_{\nu}} h_{\ell}^{(1)}(p_{\nu} r_0) T_{i\nu}^{(\ell)}. \quad (6)$$

Accordingly, we must define

$$Z_{\alpha k}^{(\ell)}(E) = g_{\alpha k} h_{\ell}^{(1)}(p_k r_0) \sqrt{\mu_k p_k}. \quad (7)$$

In the latter equations, j_{ℓ} and $h_{\ell}^{(1)}$ stand for the spherical Bessel function and Hankel function of the first kind, respectively. These are smooth functions of the total CM energy, just like μ_k and p_k , which are the reduced mass and relative linear momentum of the two-meson system in the k -th channel, respectively. The constants $g_{\alpha k}$ stand for the intensities of the $q\bar{q} \rightarrow MM$ couplings. A distance scale ~ 0.6 fm (for light quarks) is represented by r_0 . In the text we have stripped Z of a reference to ℓ and α .

Note, moreover, as can be easily seen from expressions (2) and (6), that the singularity structures of the production and scattering amplitudes are identical, since $\Re(Z_k)$, which is proportional to the spherical Bessel function in Eq. (6), is a smooth function of the total invariant mass.

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